Car accidents on a single-lane highway

Ding-wei Huang and Yu-ping Wu
Department of Physics, Chung Yuan Christian University, Chung-li, Taiwan
(Received 6 August 2000; published 10 January 2001)

We study the occurrence of car accidents in the Nagel-Schreckenberg model. Both the effects of stochastic braking and speed limit are analyzed. An approximate scaling relation is observed with the varying of speed limit. The stochastic noise will enhance the probability for accidents in the low density region and suppress that in the high density region. The probability for car accidents to occur becomes a broadened distribution over a wide range of density.

DOI: 10.1103/PhysRevE.63.022301 PACS number(s): 45.70.Vn

I. INTRODUCTION

The cellular automaton approach to traffic flow has attracted much interest recently [1]. Compared to other approaches in modeling traffic flow, cellular automaton models can be used very efficiently for computers to perform real time simulations [2]. As the underlying dynamics is governed by a few simple update rules, this approach also allows the flexibility to adapt complicated features observed in real traffic. More recently, the occurrence of car accidents has been studied within this framework. In Ref. [3], the numerical works have been reported. In Ref. [4], the exact results are analyzed. However, the speed limit has been set too low and the stochastic driving behavior has not been considered. Thus results an unrealistic feature: the accidents occur only when the car density is high; at low density, there are no accidents observed. In real traffic, accidents also occur in the low density region. When the density is low, the speed is high. Careless driving could easily result in an accident. If one considers the occurrence of car accidents per car, instead of per road, there might not be much difference between the low density and high density regions.

Thus it would be interesting to study this problem with a general spectrum. In this paper, we study the occurrence of car accidents in a cellular automaton model. Both the effects of stochastic braking and speed limit will be analyzed. The accidents resulting from not keeping the safety distance and speeding will also be discussed.

II. CAR ACCIDENTS

The Nagel-Schreckenberg model is a basic model of traffic flow on a single-lane highway [5]. The road is divided into L cells. Each cell can be either empty or occupied by a car with an integer speed $v \in \{0,1,\ldots,v_{max}\}$, where v_{max} is the speed limit. With periodic boundary condition, the number of cars is conserved. At each time step, the configuration of N cars is updated by the following four rules, which are applied in parallel to all cars. The first rule is the acceleration. If the speed of a car is lower than v_{max}, the speed is advanced by one. The second rule is the slowing down due to other cars. If a car has d empty cells in front of it and a speed larger than d, the speed is reduced to d. The third rule is the randomization, which introduces a noise to simulate the stochastic driving behavior. The speed of a moving car ($v \geqslant 1$) is decreased by one with a braking probability p_1. In the fourth rule, the position of a car is shifted by its speed v. Iterations over these simple rules already give realistic results. The model contains three parameters: the maximum speed v_{max}, the braking probability p_1, and the average density $\rho = N/L$. The real traffic data can be well described by parameters $v_{max}=5$ and $p_1=0.5$, where the length of a cell is 7.5 m and one time step corresponds to approximately 1 s [5].

In the basic model, car accidents will not occur. The second rule of the update is designed to avoid accidents; the driving scheme respects the safety distance. In real traffic, car accidents occur most likely when drivers do not respect the safety distance, which often happens when the car ahead is moving. If a moving car is suddenly stopped, careless driving of the following car will result in an accident. In Ref. [3], an approximate probability for an accident to occur is proposed. When the following three conditions are satisfied a car will cause an accident with a probability p_2. The first condition is $d \leqslant v_{max}$, which means the position of the car ahead can be reached by the next time step. The second condition is a moving car ahead. The third condition is that the moving car ahead stops at the next time step. The occurrence of car accidents is proportional to the probability p_2. In the following, $p_2 = 0.1$ is simply assumed. The probability per car and per time step for an accident to occur is denoted by P_{ac}. In Ref. [3], the value of P_{ac} is studied in the special case of $v_{max}=3$ and $p_1=0$. In the next section, we study the dependence of P_{ac} on both v_{max} and p_1. We also note that the first condition of accidents, $d \leqslant v_{max}$, presumes that both the safety distance and the speed limit were not respected. With $d = v_{max}$, the position of the car ahead can only be reached at the next time step by a car with a speed larger than the speed limit, i.e., $v = v_{max}+1$. These two issues can be easily separate. The effects of speeding can be excluded by replacing the condition with $d < v_{max}$. The differences will also be discussed.

III. NUMERICAL RESULTS

First, we study the dependence of speed limit v_{max}. The stochastic noise is neglected, i.e., $p_1 = 0$. Car accidents will not occur until the density reaches a critical value. With the increase of density ρ, the value of P_{ac} increases, reaches a maximum, and decreases with further increase of ρ. The nu-

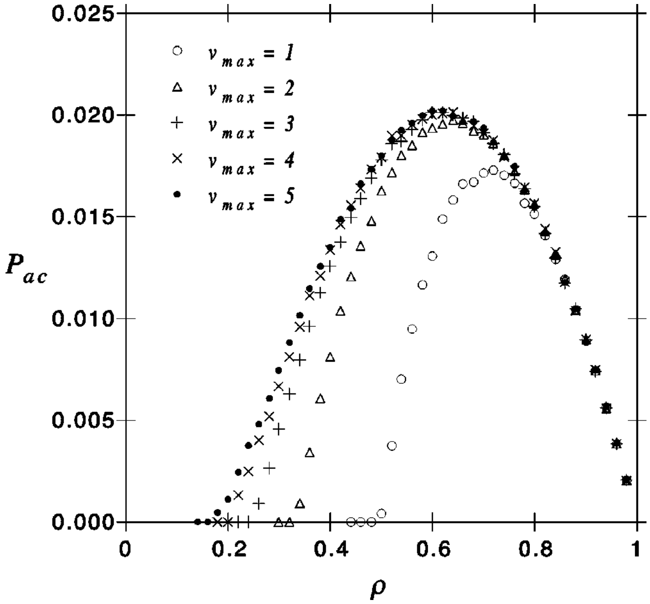

FIG. 1. Probability P_{ac} as a function of density ρ for various v_{max}. The braking probability $p_1=0$. The data are obtained with a system of $L=10^3$; an average over 500 time steps and 50 initial configurations is taken for each data point.

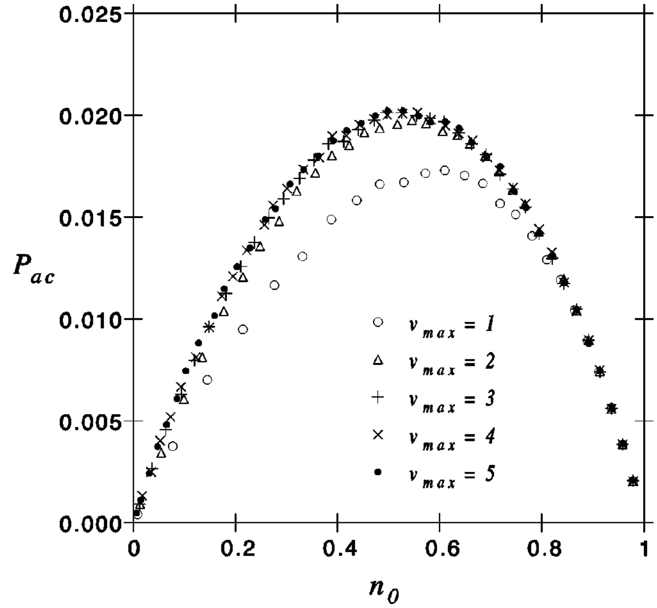

FIG. 2. Probability P_{ac} as a function of stopped car fraction n_0 for various v_{max}. The braking probability $p_1=0$.

merical results are shown in Fig. 1. The density for the onset of P_{ac} is known as the critical density, below which no accidents will occur. The density for the maximum of P_{ac} is known as the most probable density, at which accidents occur most frequently.

As prescribed in the third condition of an accident, the occurrence of a car accident is related directly to the number of stopped cars. Below the critical density, there is no stopped cars and therefore no accidents. Near the critical density, the value of P_{ac} is proportional to the fraction of stopped cars n_0. As further prescribed in the second condition of an accident, only the suddenly stopped car will cause an accident. Thus in the high density region, where n_0 also assumes a large value, the traffic flow becomes a stop-and-go wave and the value of P_{ac} decreases linearly with the increase of n_0. The results are shown in Fig. 2.

As the speed limit v_{max} increases, the probability for the occurrence of car accidents increases as expected, especially in the low density region. When the density is high, the speed limit is irrelevant and a scaling relation is expected. A similar behavior has also been observed in the fundamental diagram, i.e., the flow versus density. With increasing v_{max}, both the critical density and the most probable density shift toward the low density region. It is interesting to note that the onset of P_{ac} is smoother in the case with a higher speed limit, which also reflects a similar behavior in the onset of n_0. An approximate scaling between P_{ac} and n_0 can also be observed in Fig. 2. The case of $v_{max}=1$ is an exception where an additional particle–hole symmetry dictates the behavior.

When the stochastic driving behavior is considered, the value of P_{ac} is enhanced in the low density region and suppressed in the high density region. The results are shown in Fig. 3. The fraction of stopped cars n_0 increases with the increase of braking probability p_1. In the low density region, increasing n_0 leads to a larger value of P_{ac}. On the contrary, in the high density region, increasing n_0 leads to a lower value of P_{ac}. Thus the distribution $P_{ac}(\rho)$ is broadened over a much wider range of density.

The critical density decreases with the increase of p_1. Near the critical density, the value of P_{ac} is still proportional to n_0. However, the proportional constant decreases with the increase of p_1. The results are shown in Fig. 4. With a large braking probability p_1, drivers are apt to drive slower. In a certain way, the driving is more careful. With the same fraction of stopped car n_0, the occurrence of car accidents is

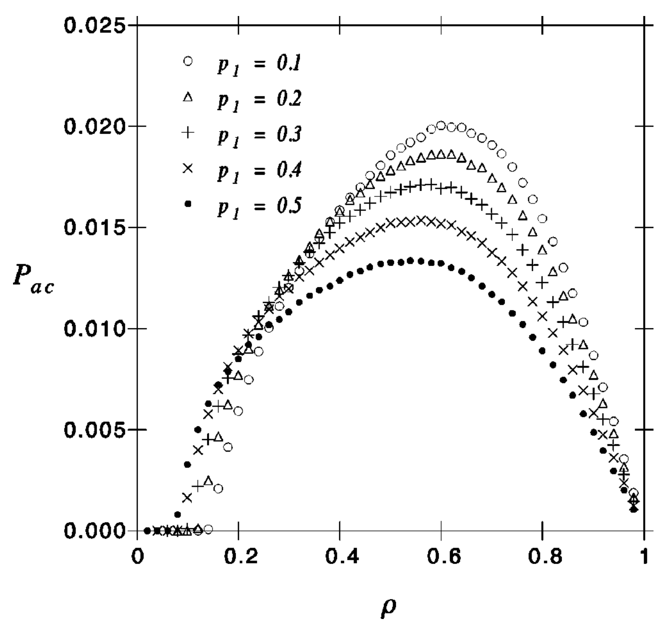

FIG. 3. Probability P_{ac} as a function of density ρ for various p_1. The speed limit $v_{max}=5$.

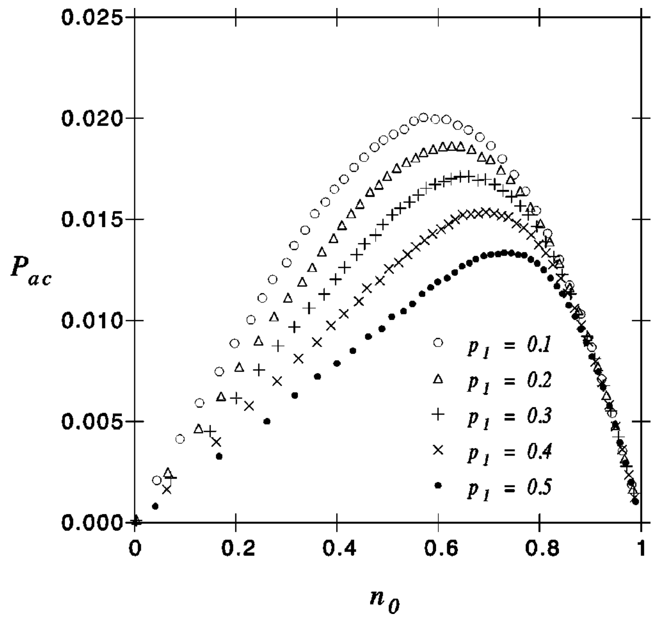

FIG. 4. Probability P_{ac} as a function of stopped car fraction n_0 for various p_1. The speed limit $v_{max}=5$.

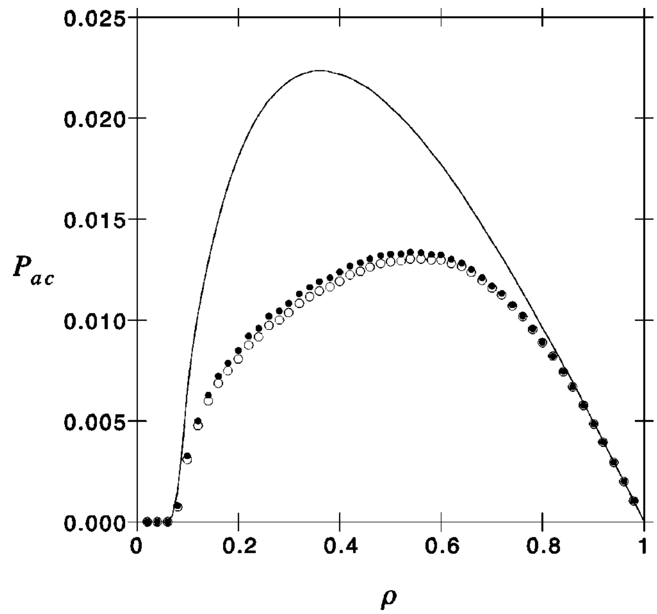

FIG. 5. Probability P_{ac} as a function of density ρ. The speed limit $v_{max}=5$ and the braking probability $p_1=0.5$. The closed circles are the results of $d \leqslant v_{max}$; the open circles are the results of $d < v_{max}$, which exclude the effects of the speeding. The solid line is the mean-field result.

suppressed. However, the number of stopped cars increases with the increase of p_1. With the same car density ρ, the occurrence of car accidents is enhanced. The scaling relation between P_{ac} and n_0 can still be observed in the very high density region, where most cars are stopped. It is interesting to note that the scaling relation shown in Fig. 2 is due to the irrelevance of speed limit v_{max} in the high density region. There is also a scaling relation between P_{ac} and ρ with different v_{max}, see Fig. 1. However, the braking probability p_1 is important in the high density region. There is no scaling between P_{ac} and ρ with different p_1, see Fig. 3.

We observe that only a small fraction of accidents result from speeding, i.e., $d=v_{max}$ in the first condition of accidents. The results are shown in Fig. 5. This can be readily understood from the approximate scaling relation between P_{ac} and n_0 with different v_{max}.

Neglecting correlations, an analytical expression for the probability P_{ac} can be obtained within the mean-field theory as

$$P_{ac}=\frac{p_2}{\rho}\left\{\sum_{i=0}^{v_{max}}(1-\rho)^i\right\}\{\rho(1-n_0)\}\{\rho n_0\}, \qquad (1)$$

where the factors in three braces correspond to the three conditions of accidents, respectively. The result are shown in Fig. 5. The mean-field results approach the data around the critical density and also in the very high density limit. In between these two densities, the mean-field theory overestimates the value of P_{ac}. As the mean-field theory neglects correlations, it will give meaningful results only when the value of P_{ac} is small, i.e., the onset of P_{ac} and the very high density limit. Basically, the occurrence of car accidents cannot be well-described without considering the correlations, both spatial and temporal.

IV. CONCLUSION

In this paper we study the occurrence of car accidents in a traffic model. The probability per car for an accident to occur (P_{ac}) is related to the fraction of stopped cars on the road (n_0). In the low/high density region, the value of P_{ac} increases/decreases with the increase of n_0. The effects of the braking probability p_1 and the speed limit v_{max} are analyzed. Without stochastic noise, the increase of v_{max} will

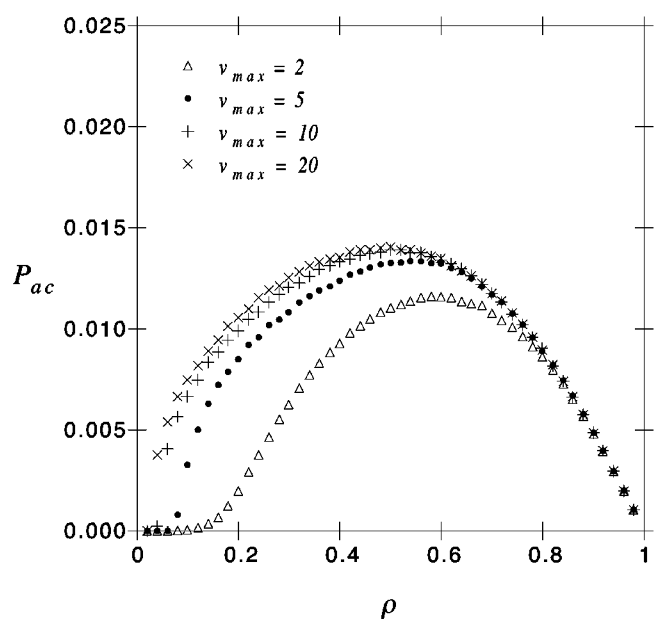

FIG. 6. Probability P_{ac} as a function of density ρ for various v_{max}. The braking probability $p_1=0.5$.

soften the onset of P_{ac}. On the contrary, with stochastic noise, the increase of v_{max} will steepen the onset of P_{ac}. The critical density decreases with the increase of both stochastic noise and speed limit; the most probable density also shows the same feature. The maximum of P_{ac} increases with the increase of speed limit, but decreases with the increase of stochastic noise. The stochastic driving behavior will enhance/suppress the value of P_{ac} in the low/high density region. Thus the distribution $P_{ac}(\rho)$ is broadened.

A higher speed limit will enhance P_{ac} for all density. However, the effects decrease with the increase of v_{max}. An approximate scaling relation between P_{ac} and n_0 is observed with various values of v_{max}. The exception of the case $v_{max}=1$ is related to the particle–hole symmetry, which is absent for $v_{max}>1$. With this scaling, the value of P_{ac} will increase only a little by further increasing v_{max}, see Fig. 6. Thus instead of speeding, most car accidents are the result of not keeping the safety distance.

Basically, both spatial and temporal correlations are important in the phenomena of car accidents. Thus the distribution $P_{ac}(\rho)$ cannot be well described by the mean-field theory. When the value of P_{ac} is small, i.e., near the critical density and in the very high density limit, the mean-field results are satisfactory. An analytical approach including both spatial and temporal correlations is desired and left for future study.

[1] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).

[2] J. Wahle, L. Neubert, and M. Schreckenberg, Comput. Phys. Commun. **121-122**, 402 (1999).

[3] N. Boccara, H. Fukś, and Q. Zeng, J. Phys. A **30**, 3329 (1997).

[4] D.W. Huang, J. Phys. A **31**, 6167 (1998).

[5] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992); M. Schreckenberg, A. Schadschneider, K. Nagel, and N. Ito, Phys. Rev. E **51**, 2939 (1995).